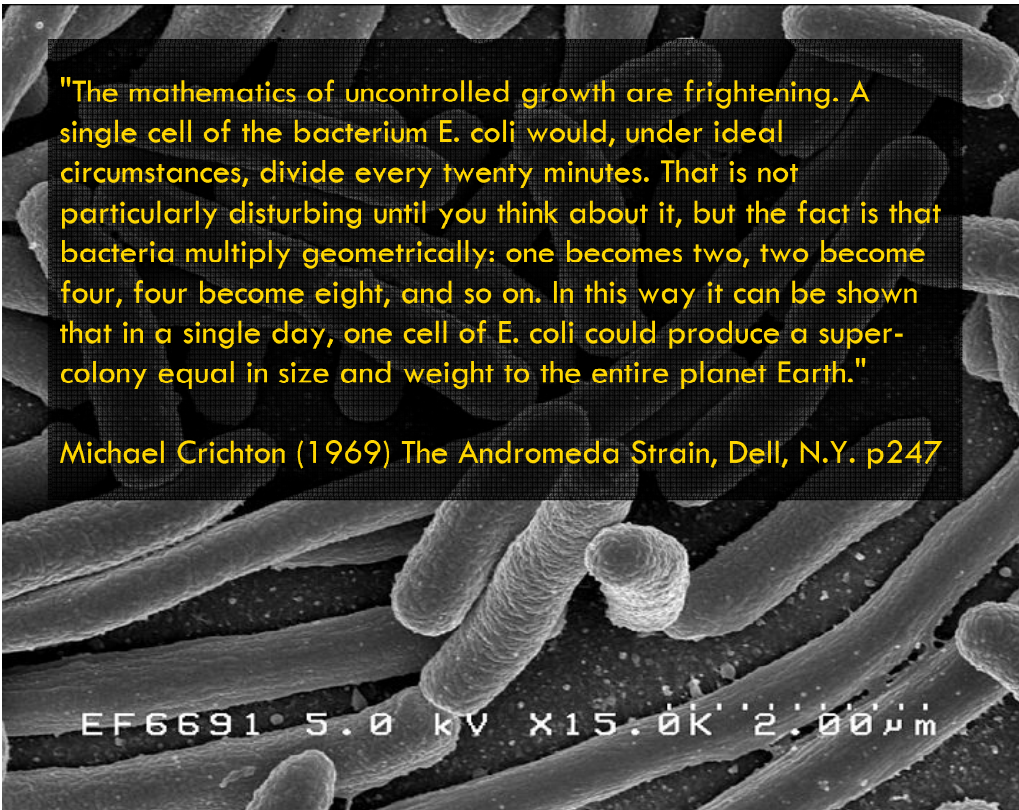




homework  
questions?

"people, history, darwin, and  
apocalypse"



"The mathematics of uncontrolled growth are frightening. A single cell of the bacterium E. coli would, under ideal circumstances, divide every twenty minutes. That is not particularly disturbing until you think about it, but the fact is that bacteria multiply geometrically: one becomes two, two become four, four become eight, and so on. In this way it can be shown that in a single day, one cell of E. coli could produce a super-colony equal in size and weight to the entire planet Earth."

Michael Crichton (1969) *The Andromeda Strain*, Dell, N.Y. p247

Population growth:

$$P(t) = P_0 e^{rt}$$

**NOTE:** your book uses  $P_0 e^{kt}$

Population growth:

$$P(t) = P_0 e^{rt}$$

population at time  $t$

time

population at initial time  $t=0$

exponential growth rate

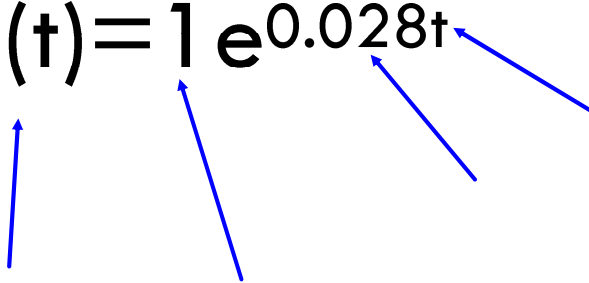
**Why people are like money earning interest**

## **Why people are like money earning interest**

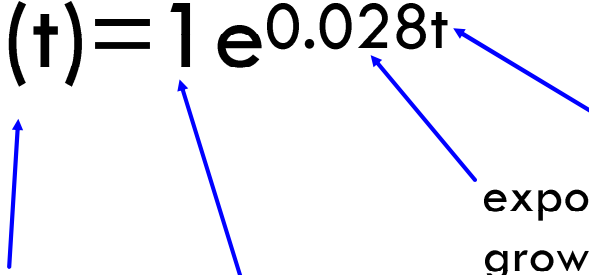
Money over time produces more money, which produces more money on that money, which produces more money on that money, etc. This process happens continuously.

People over time produce more people, who produce more people, who produce more people, who produce more people, etc. This process happens continuously.

In 1700, there were about 1 million people in the US. Benjamin Franklin in 1751 noted that the population seemed to double every 25 years.

$$P(t) = 1 e^{0.028t}$$


$P(t)$  is in million of people

$$P(t) = 1 e^{0.028t}$$


population at time  $t$       population in 1700      exponential growth rate      years after 1700

$P(t)$  is in million of people

$$P(t) = 1 e^{0.028t}$$

Questions:

1. What will the population be in 1800?
2. When will the population be 100 million?

$$P(t) = 1 e^{0.028t}$$

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$$P(t) = 1 e^{0.028t}$$

Questions:

2. When will the population be 100 million?

How did we get the  $k = 0.028$  in the equation above?

$$P(t) = P_0 e^{rt}$$

$$P(t) = P_0 e^{rt}$$

$$2P_0 = P_0 e^{r25}$$

$$2 = e^{r25}$$

$$\ln 2 = 25r \ln e$$

$$\ln 2 = 25r$$

$$\ln 2 / 25 = r$$



Because of this technique,  
if we know the doubling  
time, we can always find **r**:

$$\mathbf{r} = (\ln 2) / \mathbf{T}$$

[T is the doubling time]

Rearranging  
 $\mathbf{r} = (\ln 2) / \mathbf{T}$

we can also find the  
doubling time if we know **r**:

$$\mathbf{T} = (\ln 2) / \mathbf{r}$$

Quick check:

In 2002, the population of India was about 1034 million and the exponential growth rate was 1.4% per year.

- (a) What is the equation of the population?
- (b) What is the period doubling time?

Quick check:

In 2002, the population of India was about 1034 million and the exponential growth rate was 1.4% per year.

- (a) What is the equation of the population?

$$P(t) = 1034e^{0.014t}$$

[t is the number of years since 2002, P(t) is in millions]

- (b) What is the period doubling time?

$$T = (\ln 2) / 0.014 \approx 49.51$$

### REALITY CHECK:

In 1990, the population of NYC was 7.323 million.  
In 2000, the population of NYC was 8.008 million.

Find the equation of growth, and the population doubling time.

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In 1990, the population of NYC was 7.323 million.  
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Find the equation of growth, and the population doubling time.

$$P(t) = 7.323e^{rt}$$

$$P(10) = 8.008 = 7.323e^{10r}$$

$$8.008/7.323 = e^{10r}$$

$$1.0935 = e^{10r}$$

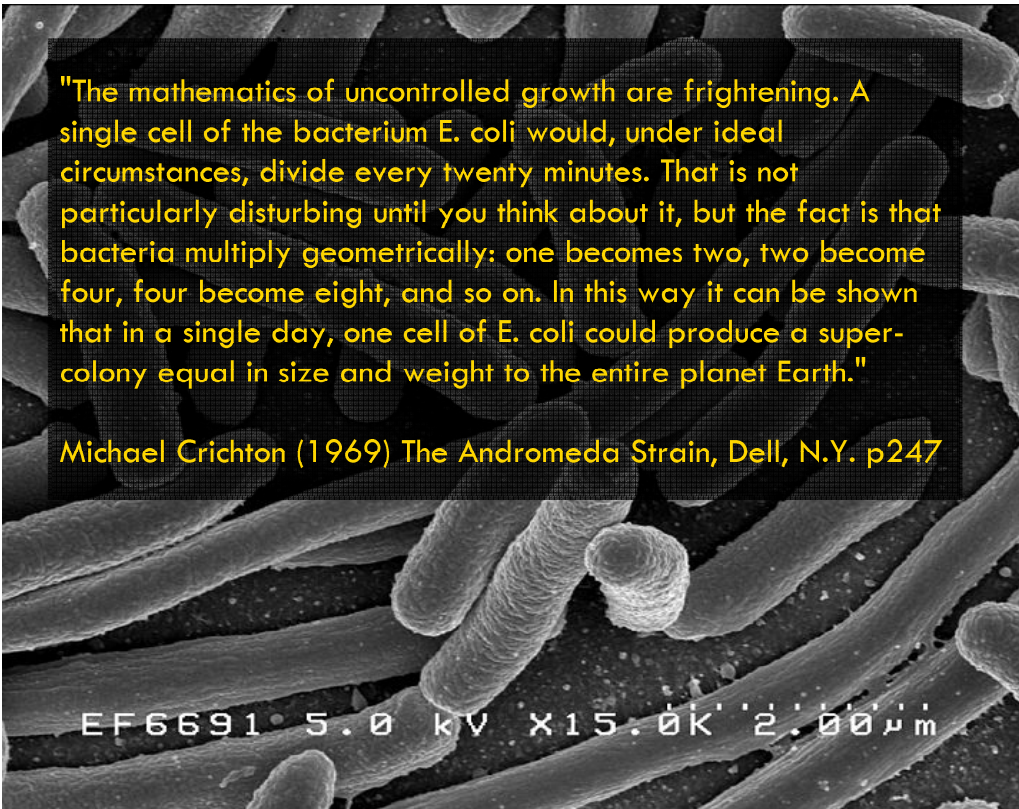
$$\ln(1.0935) = 10r$$

$$r = \ln(1.0935)/10 = 0.0089$$

So  $P(t) = 7.323e^{0.0089t}$  is the equation

And the doubling time is  $(\ln 2)/0.0089$ , which is about 77 years.

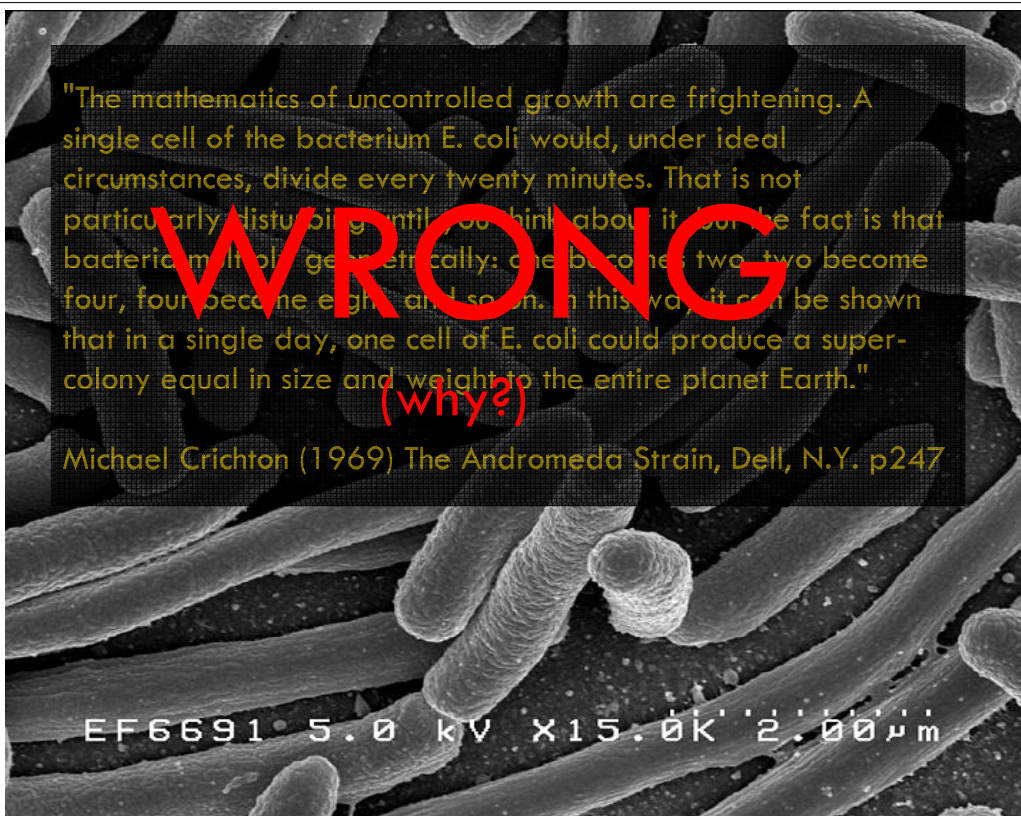
logistic equation  
logistic equation  
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logistic equation  
logistic equation



"The mathematics of uncontrolled growth are frightening. A single cell of the bacterium E. coli would, under ideal circumstances, divide every twenty minutes. That is not particularly disturbing until you think about it, but the fact is that bacteria multiply geometrically: one becomes two, two become four, four become eight, and so on. In this way it can be shown that in a single day, one cell of E. coli could produce a super-colony equal in size and weight to the entire planet Earth."

Michael Crichton (1969) The Andromeda Strain, Dell, N.Y. p247

EF6691 5.0 kV X15.0K 2.00um



You do not need to memorize!

**logistic function:**

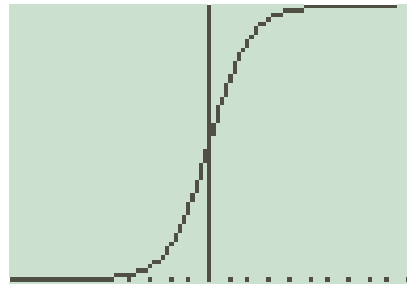
$$P(t) = \frac{a}{1 + be^{-kt}}$$

**logistic function:**

$$P(t) = \frac{1}{1 + 1e^{-1t}}$$

What does it look like if the constants are all 1?

```
Plot1 Plot2 Plot3
\Y1=1/(1+1e^(-1X
))
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```



CHECK: A ship carrying 1000 passengers gets shipwrecked on an island. The natural resources of the island limit the population to 5780. The equation is:

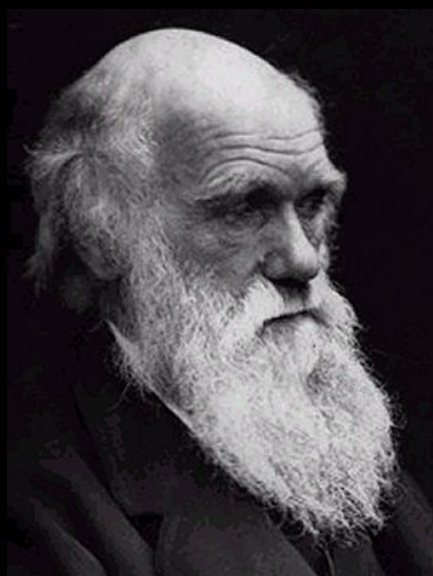
$$P(t) = \frac{5780}{1 + 4.78e^{-0.4t}}$$

A) Graph the function

B) Find the pop after 0, 1, 2, 5, 10, and 20 years



malthus



darwin

HW: Section 4.6#1-4, 7, 15.



