Multivariable Calculus: Problem Set 2A

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Instructions: You are encouraged to discuss general strategies to approach the questions on this problem set with your classmates, but you must work and write up your solutions to the problems entirely on your own. Of course, you are always welcome to meet with me to talk about any question you are having difficulty with. Please pay attention to making your solutions as clear as possible for the reader; mathematical communication is an important skill that you will develop in this course.

Problem 1a

Show that the graph of

$$\vec{r}(t) = \sin(t)\vec{i} + 2\cos(t)\vec{j} + \sqrt{3}\sin(t)\vec{k}$$

is a circle, and find its center and radius. [Hint: Show that the curve lies both on a sphere and a plane.]

Problem 1b

Consider the curve $\vec{r}(t) = 2\cos(5t)\vec{i} + \sin(3t)\vec{j} + ct\vec{k}$, with c being a constant. If the height of the function (the z-value) is 30 after 10 revolutions, what is the value of c?

Problem 2

Prove $\frac{d}{dt}[f\vec{r}] = f\frac{d\vec{r}}{dt} + \frac{df}{dt}\vec{r}$, where f is a function of t and $\vec{r}$ is a vector-valued function of t.

Problem 3

(a) Find the points where the curve $\vec{r}(t) = t\vec{i} + t^2\vec{j} - 3t\vec{k}$ intersects the plane $2x - y + z = -2$.

(b) For the curve and the plane in part (a), find, to the nearest degree, the acute angle that the tangent line to the curve makes with a line normal to the plane at each point of intersection.

Problem 4

On page 876 in your textbook, the authors write:

Quotation: Graphs of vector-valued functions range from continuous and smooth to discontinuous and wildly erratic. in this text we will not be concerned with graphs of the latter type, so we will need to impose restrictions to eliminiate the unwanted behavior. We will say that $\vec{r}(t)$ is *smoothly parametrized* or that $\vec{r}(t)$ is a *smooth function* of t if $\vec{r}'(t)$ is continuous and $\vec{r}'(t) \neq \vec{0}$ for any allowable value of t.

This is a pretty important statement. Come up with an $\vec{r}(t)$ that is not $\vec{r}(t) =< constant, constant, constant >$ such that for at least one value of t, $\vec{r}'(t) = \vec{0}$. Describe how the tangent vector evolves over time (you may use diagrams as well), and why the book might not want to consider your vector-valued function.

Problem 5

Show that in cylindrical coordinates a curve given by the parametric equations $r = r(t)$, $\theta = \theta(t)$, $z = z(t)$ for $a \leq t \leq b$ has arc length:

$L = \int \sqrt{(\frac{dr}{dt})^2 + r^2(\frac{d\theta}{dt})^2 + (\frac{dz}{dt})^2} dt$

(You're going to have to remember how to calculate arc length from your course last year!)

When you've done that, find the arc length of the curve $r = t^2$, $\theta = \ln(t)$, $z = \frac{1}{3}t^3$ where $1 \leq t \leq 2$.

(For bonus points – mainly because I haven't done this yet – use winplot and your knowledge of cylindrical and rectangular coordinates to graph this segment. Explain how you did it!)