# Multivariable Calculus: Problem Set 2B

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Instructions: You are encouraged to discuss general strategies to approach the questions on this problem set with your classmates, but you must work and write up your solutions to the problems entirely on your own. Of course, you are always welcome to meet with me to talk about any question you are having difficulty with. Please pay attention to making your solutions as clear as possible for the reader; mathematical communication is an important skill that you will develop in this course.

### Problem 1

Let $\vec{r}(t) =< a\cos(t), a\sin(t), ct >$where $a$ and $c$ are both non-zero.

First: Use the formula $\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$ to find $\vec{B}(t)$.

Second: Check your answer by using the (unproven) formula $\vec{B}(t) = \frac{\vec{r}'(t)\times\vec{r}''(t)}{||\vec{r}'(t)\times\vec{r}''(t)||}$.

[Because I haven't yet done this, and the book skips this, double bonus points and glory goes to you if you prove this (unproven) formula!]

### Problem 2

Use $\kappa(t) = \frac{||\vec{T}'(t)||}{||\vec{r}'(t)||}$ to show that in 2-space the curvature of a smooth parametric curve $< x(t), y(t) >$ is:

$$\kappa(t) = \frac{|x'y'' - y'x''|}{(x'^2 + y'^2)^{3/2}}$$

where primes denote differentiation with respect to $t$.

### Problem 3

Choose and solve *one* of the following (problems 51 and 52 in Section 13.5):

(51) Find the maximum and minimum values of the radius of curvature for the curve $< \cos(t), \sin(t), \cos(t) >$.

(52) Find the minimum value of the radius of curvature for the curve $< e^t, e^{-t}, \sqrt{2}t >$.

### Problem 4

The nuclear accelerator at the Enrico Fermi Laboratory is circular with a radius of 1 km. Find the scalar normal component of acceleration of a proton moving around the accelerator with constant speed of $2.9 \times 10^5$ km/s.