Name:_____ Band:_____

Calculus | Packer Collegiate Institute

DOUBLE CHALLENGE PROBLEMS for Basic Derivatives

Hi all. Here are some hard problems. Some I know how to solve. Some I don't know how to solve. I want you to play around with the problems that interest you!

You have to do neat, *formal write-ups* for only 2 of the problems. So play around with a few of them and find the ones you are most confident about and work though them! What I expect in a formal write-up is more than a bunch of equations. I want you to explain each key step you take in words. For example, when we were asked to where a particular function had a horizontal tangent line, each of you figured out you needed to find when the derivative equaled zero. Explain that! Also, draw pictures! Sketch what's on your calculator (and give the window you used to graph it)!

If you can't get the answer, still turn in a formal write-up. Say what you tried, why you tried it, and why it didn't work.

This is an exercise in problem solving AND mathematical communication.

1. Using your graphing calculator and your brains, find where $f(x) = xe^x$ and $g(x) = 15x^{\sqrt{2}} + x^2 + \pi x$ have parallel tangent lines. (Round your answer to three decimal points.) Explain each step of your calculations in words, and also how and where you used your calculator.

2. Determine coefficients a, b, c, and d such that $f(x) = ax^3 + bx^2 + cx + d$ satisfies $f(1) = 1$, $f(-1) = 5$, $f'(2) = 2$, and $f''(-1) = 3$.

3. Where (at which x values) do the functions $f(x) = 2x^2 + 5x - 3$ and $g(x) = 5x^2 - 2x - 13$ have perpendicular tangent lines?

4. a. If $f'(x) = \frac{1}{\sqrt{x}} + 4x + 15$, can you find what $f(x)$ is? In fact, there are multiple answers for $f(x)$s! Can you find another one?

   b. If $f''(x) = e^x(x^2 + 4x + 2)$, can you find what $f(x)$ is? In fact, there are multiple answers for $f(x)$s! Can you find another one?

5.a. Find $\frac{d}{dx}[e^{2x}]$ b. Find $\frac{d}{dx}[e^{3x}]$ c. Find $\frac{d}{dx}[e^{4x}]$ d. Find $\frac{d}{dx}[e^{nx}]$ (where n is a positive integer)

[For (d) if you know about *proof by induction*, that might help you!]

6. a. Find $\frac{d}{dx}[(x^7 + 2x - 3)^3]$

Hint: Apply the product rule twice to expand and simplify: $\frac{d}{dx}[fff]$. In other words, you are coming up with your own product rule, but for the product of *three* functions.

b. Find $\frac{d}{dx}[(x^2 + 1)^{50}]$

7. a. Define what it should mean to say that two curves intersect at right angles.

b. Prove that the curves $y = \frac{1}{x}$ and $y = \frac{1}{2 - x}$ intersect at right angles.