Problem Solving Lesson Plan

Part I: Playing with Weights

Objective: For this part, you should familiarize yourself with the "forward weight problem," gain a concrete understanding of what the problem is asking, and recognize patterns.

The Problem: [Work with partner.] You are a goldsmith and day in and day out, customers bring you jewelry to weigh. This jewelry is usually marked with the weight, such as 15 ounces. However, you have gotten cheated in the past (!) and you don't want to believe what is marked on the jewelry.

Unfortunately all you have is an old school balance (as seen to the left) and four weights. The weights are exactly 1 ounce, 2 ounces, 17 ounces, and 20 ounces.

To check the veracity of the weight of jewelry brought to you, you're going to have to use these four weights and the scale!

Question 1: A customer brings you a ring which is marked 3 ounces. How can you use the scale to check if the ring is indeed 2 ounces?

Question 2: A customer brings you a ring which is marked 4 ounces. How can you use the scale to check if the ring is 4 ounces?

Question 3: A customer brings you a bracelet which is marked 41 ounces. How can you use the scale to check if the ring is 41 ounces.

Generalizing! [Work with partner.]

Make a table showing how you would test pieces of jewelry with different weight labels from 1 ounce to 40 ounces.

1 ounce		21 ounces	
2 ounces		22 ounces	
3 ounces		23 ounces	
4 ounces		24 ounces	
5 ounces		25 ounces	
6 ounces		26 ounces	
7 ounces		27 ounces	
8 ounces		28 ounces	
9 ounces		29 ounces	
10 ounces		30 ounces	
11 ounces		31 ounces	
12 ounces		32 ounces	
13 ounces		33 ounces	
14 ounces		34 ounces	
15 ounces		35 ounces	
16 ounces		36 ounces	
17 ounces		37 ounces	
18 ounces		38 ounces	
19 ounces		39 ounces	
20 ounces		40 ounces	

Question 4: Now that you are done, compare your chart with another group's chart. Did you get the exact same thing or did you differ on some? If you differed are you both correct?

Question 5: What weights couldn't you verify? Why couldn't you verify them?

Question 6 [as a whole class]: What observations or patterns did you notice? Right now you're just *brainstorming* – write anything you noticed or thought, even if it seems irrelevant (or stupid) now.

Part II: Working Backwards

Objective: In this section, you will be introduced to the "backwards weight problem," start applying your observations and thoughts, and doing a bit of guessing and checking and refining your ideas.

The Problem: Instead of being given 4 weights and finding out what jewelry weights you can verify, in this problem, you are given what jewelry weights you *can* verify and you have to determine the 4 weights you have.

So here's the problem. You're a goldsmith and unfortunately you have only a single 40 ounce weight and old school balance. Even worse, you clumsily dropped the weight and it cracked and split into four weights (with integer ounce weights). But strangely enough, with those four weights, you are now able to verify all the jewelry that comes your way from 1 ounce to 40 ounces.

What are the 4 weights that the 40 ounce weight split into?

Things to consider:

You and your partner showed that that 1 ounce, 2 ounces, 17 ounces, and 20 ounces won't solve this "backwards problem," even though they add up to 40 ounces. Think about why those 4 numbers failed?

Look at your answers to Questions 4-6. They will help you get started.

Using trial and error, and refining it, is sometimes a totally valid problem solving technique.

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Part III: Generalizing

Objective: Now that you have solved the problem, you want to see if the solution is at all "obvious." You want to look at the solution, *knowing that it is the solution*, and see if you can explain *why* that is the solution – in words. Then you want to come up with extensions for the problem.

Question 7: For the solution that you found above (1 ounce, 3 ounces, 9 ounces, and 27 ounces), write down all observations/patterns you notice in the table you filled in.

Question 8: What pattern do you notice in your solution: 1 ounce, 3 ounces, 9 ounces, and 27 ounces?

Question 9: Can you come up with an explanation for this pattern?[1]

Question 10: Extend this problem. If you could add a 5th weight of any size, which would help you weigh as many more pieces of jewelry, what would that 5th weight be? Why?

Question 11: Think of other extension questions or generalizations for this problem. You don't have answer them, just write them down.

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[1] *Optional Hint:* If you can't come up with an explanation, think of what you can measure with just the 1 ounce weight. Then think of what you can measure with just the 1 ounce and 3 ounce weight. Then think what you can measure with just the 1 ounce, the 3 ounce, and the 9 ounce weight. Then think what you can measure with all four weights. (Or, alternatively, look at the chart of the solution you produced in concert with your observations from Question 7.)