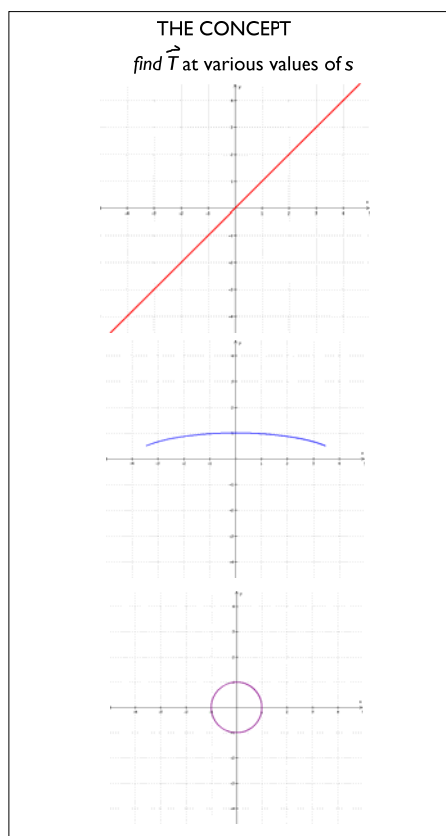


Oct 27-8:17 PM

THE CONCEPT

*imagine that we have vector  
valued functions which are  
parametrized in terms of arc  
length...*

Nov 9-7:53 PM

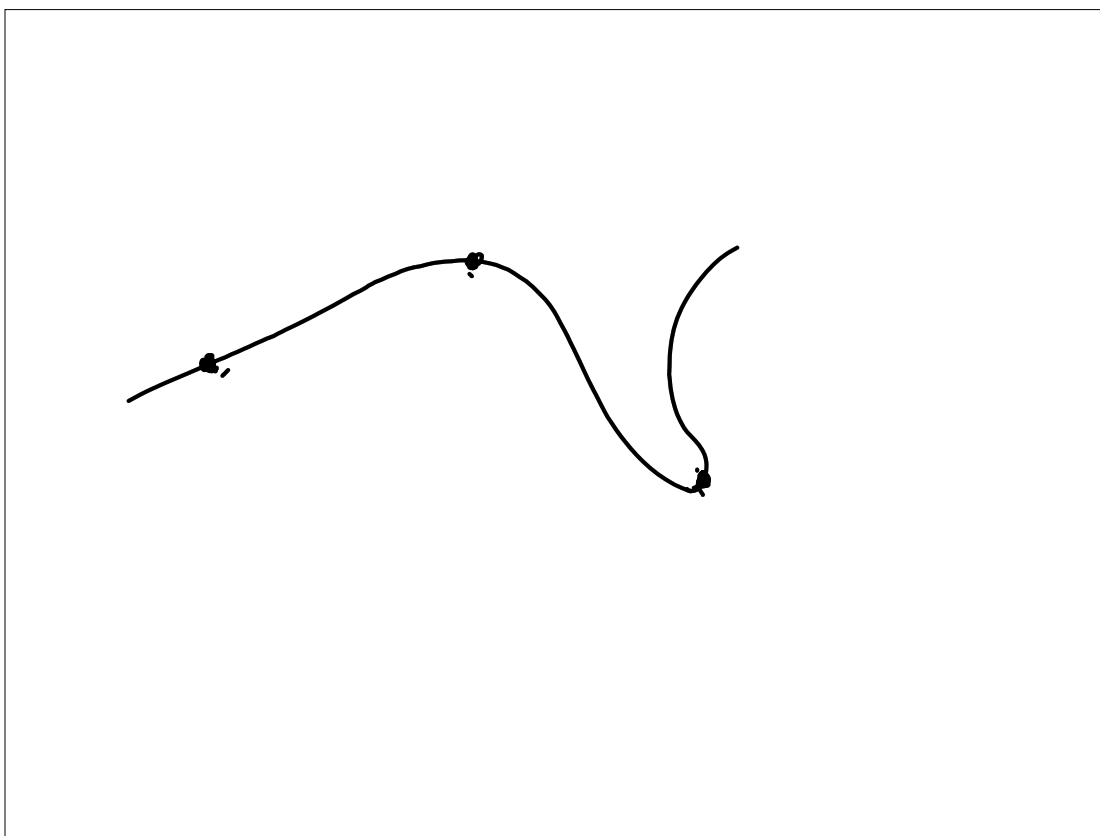


Nov 9-7:53 PM

Curvature of a function is basically how fast the direction of the tangent vector changes over time.

$$\kappa(s) = \left\| \frac{d\vec{T}}{ds} \right\| = \|\vec{r}''(s)\|$$

Nov 9-8:00 PM



Nov 14-9:34 AM

"The situation in 3-space is more complicated because bends in a curve are not limited to a single plane -- they can occur in all directions, as illustrated by

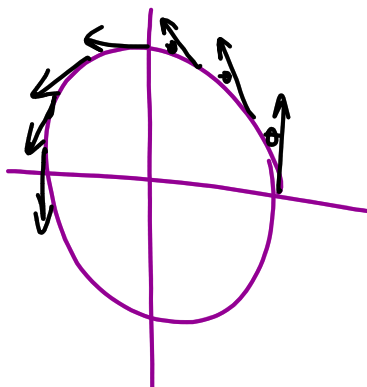


To describe the bending characteristics of a curve in 3-space completely, one must take into account  $\frac{dT}{ds}$ ,  $\frac{dN}{ds}$ , and  $\frac{dB}{ds}$ . A complete study of this topic will take us too far afield, so we will limit our discussion to  $\frac{dT}{ds}$ , which is the most important of these derivatives in applications." (Anton, 874)

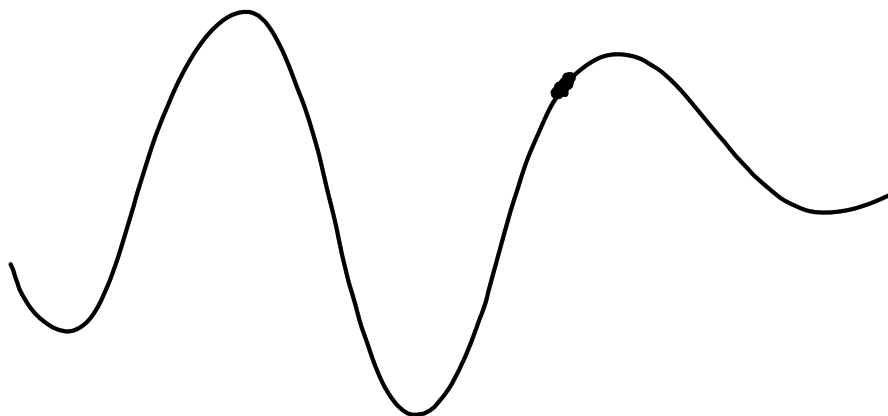
Nov 9-8:08 PM

TRUE OR FALSE:

A circle will have a constant curvature.



Nov 9-8:07 PM



Nov 14-9:42 AM

$$\|\vec{r}'(s)\| = 1 \quad \vec{r}(s) = \left\langle a \cos\left(\frac{s}{a}\right), a \sin\left(\frac{s}{a}\right) \right\rangle$$

$$\vec{r}'(s) = \left\langle -\sin\left(\frac{s}{a}\right), \cos\left(\frac{s}{a}\right) \right\rangle$$

first: does this make sense for the arc length parametrization of a circle of radius  $a$ ?

↳ equal distance in equal time

second: find the curvature of a circle of radius  $a$

Nov 9-8:05 PM

$$\vec{r}(s) = \left\langle a \cos\left(\frac{s}{a}\right), a \sin\left(\frac{s}{a}\right) \right\rangle$$

Need  $T'(s)$ ...

$$\vec{T}(s) = \vec{r}'(s) = \left\langle -\sin\left(\frac{s}{a}\right), \cos\left(\frac{s}{a}\right) \right\rangle$$

$$\vec{T}'(s) = \left\langle -\frac{1}{a} \cos\left(\frac{s}{a}\right), -\frac{1}{a} \sin\left(\frac{s}{a}\right) \right\rangle =$$

$$\|\vec{T}'(s)\| = \frac{1}{a}$$

Nov 14-9:48 AM

## formulas for curvature

$$\kappa(t) = \frac{\|\vec{T}'(t)\|}{\|\vec{r}'(t)\|}$$

$$\kappa(t) = \frac{\|\vec{r}'(t) \times \vec{r}''(t)\|}{\|\vec{r}'(t)\|^3}$$

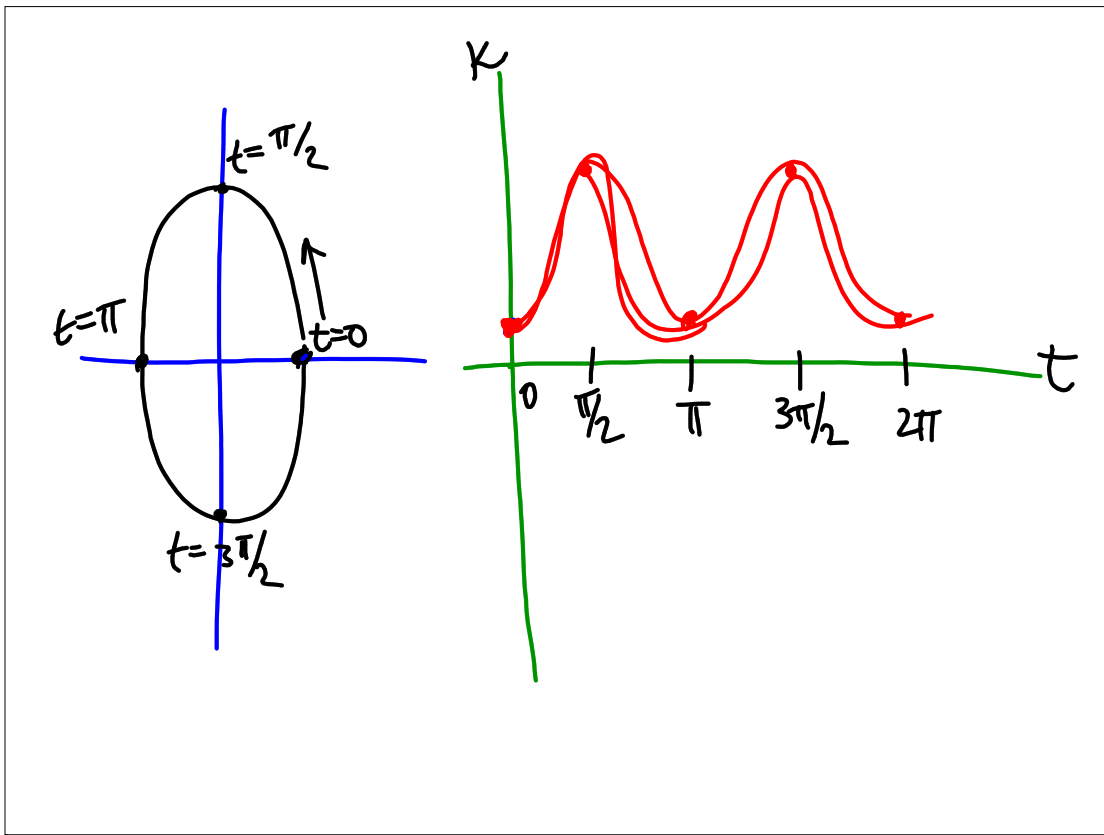
(see 893-894 for the proof)

Nov 9-8:08 PM

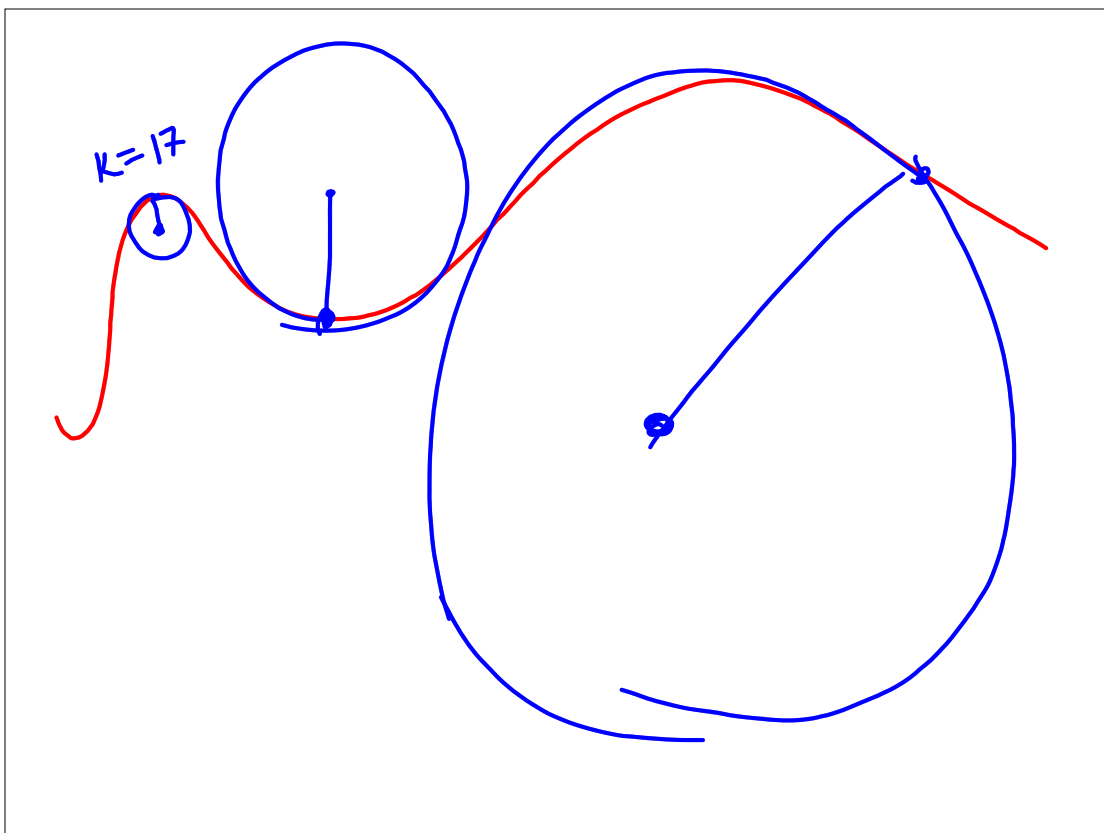
Find the curvature function for the circular helix:

$$\vec{r}(t) = \langle a \cos(t), a \sin(t), ct \rangle$$

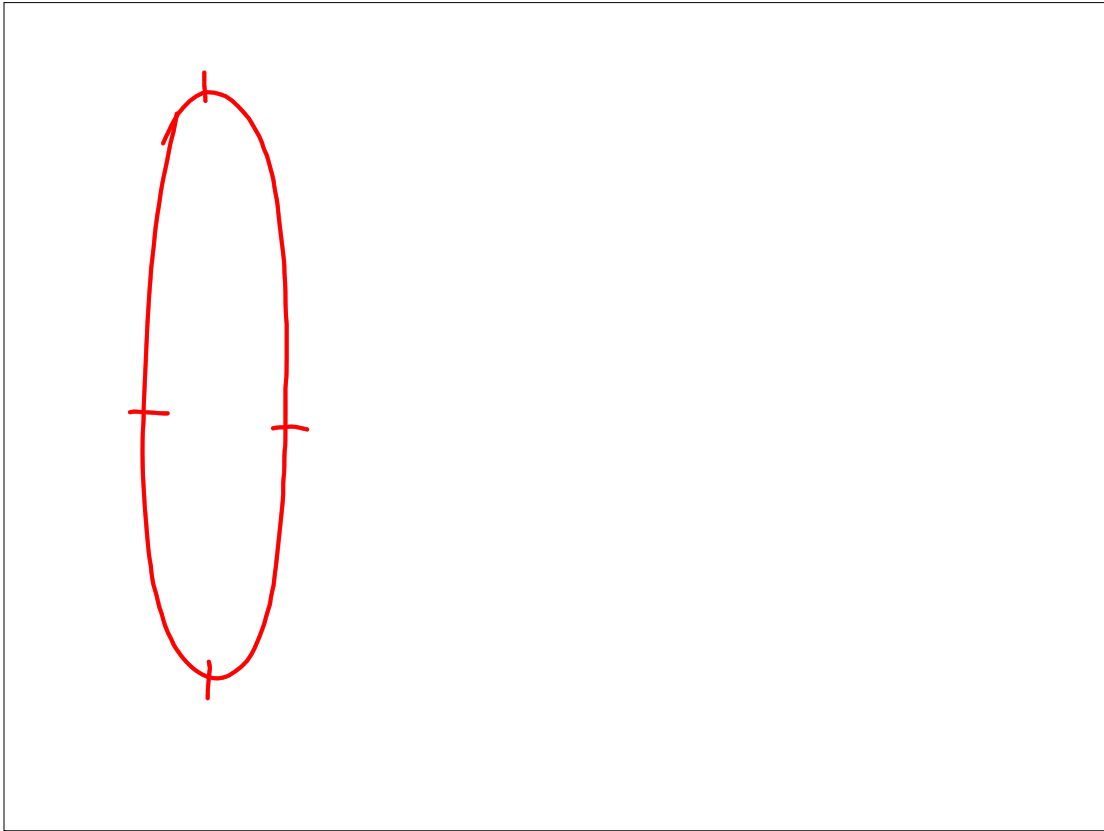
Nov 9-8:17 PM



Nov 14-9:57 AM



Nov 14-10:09 AM



Nov 14-10:07 AM

Find the curvature function for the ellipse:

$$\vec{r}(t) = \langle 2 \cos(t), 3 \sin(t) \rangle$$

Plot this curvature function on WinPlot and explain the result with reference to the original ellipse.

Nov 9-8:19 PM



## Home Enjoyment:

1. Read p. 877 on the radius of curvature
2. Section 12.5#1-4,5,6,7,9

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Oct 28-3:20 PM