MATHEMATICIAN	DATE	BAND

PUZZLES! WHAT COMES NEXT???
PRECALCULUS | PACKER COLLEGIATE INSTITUTE

Warm Up:

1. You are going to be given a set of cards. The cards have a *sequence of numbers* on them… Although there are many ways to divide up these cards, your task is to divide these cards into *three groups* based on how you figure out the next number in the sequence. (There are three different ways the next number can be found.)

In the chart below, write your categorization as well as the next number in the sequence. *The three groups may not be equal in size, and not all blanks below will be filled in!*

First Group	*Second Group*	*Third Group*
How to determine the next number in the sequence:	How to determine the next number in the sequence:	How to determine the next number in the sequence:
Letter:_____ Next #:______ Letter:_____ Next #:______ Letter:_____ Next #:______ Letter:_____ Next #:______ Letter:_____ Next #:______ Letter:_____ Next #:______	Letter:_____ Next #:______ Letter:_____ Next #:______ Letter:_____ Next #:______ Letter:_____ Next #:______ Letter:_____ Next #:______ Letter:_____ Next #:______	Letter:_____ Next #:______ Letter:_____ Next #:______ Letter:_____ Next #:______ Letter:_____ Next #:______ Letter:_____ Next #:______ Letter:_____ Next #:______

If you finish early, I have some extra cards for you to work on. They form their own group. Your job is to (a) figure out the next number in the sequences and (b) what connects these cards together to form their own group?

<u>Part I:</u> I give you the rule, you give me the first six numbers in the sequence.

1. To get any number in the sequence you add 3 to the previous number. The first number is 10.

2. To get any number in the sequence, you subtract five from the previous number. The fourth number is 10.

3. To get any number in the sequence, you multiply the previous number by 2 and then add 1. The first number is 4.

4. To get any number in the sequence, you multiply the previous number by 2 and then add 1. The third number is 3.

5. To get any number in the sequence, you add the previous number to twice the number that came before it. The first number is 7 and the second number is 3.

6. To get any number in the sequence, you multiply the previous number by 0.5. The third number is 3.

7. On a notecard provided, write down your name as well as a rule of your own that is interesting! Then generate the first six numbers for that rule. Your rule might make it on a future assessment – so I'd copy your rule and sequence of numbers in the space below!

8. To get the *n*th number in the sequence, you calculate the area of a circle with radius *n*.

9. To get the *n*th number in the sequence, you raise -1 to the power *n*.

10. To get the *n*th number in the sequence, you multiply *n* by *n-1*.

11. To get the *n*th number in the sequence, you write 1 if *n* is prime and 0 if *n* is not prime.

12. To get the *n*th number in the sequence, you multiply *n-1* by 1.3 and then add 2.

13. To get the *n*th number in the sequence, you count the number of factors of *n* and write that down (i.e. 10 has the following factors: 1, 2, 5, 10, so I would write 4 for the 10^{th} number in the sequence).

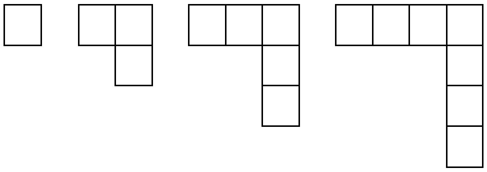
14. To get the *n*th number in the sequence, you raise 2 to the *n-1* power, and then multiply by 3.

15. To get the *n*th number in the sequence, you calculate the number of ways to choose *n-1* objects from *n+1* objects.

<u>Part II:</u> Visual Puzzles…

Puzzle #0: A Pixel Puzzle:

Puzzle #1

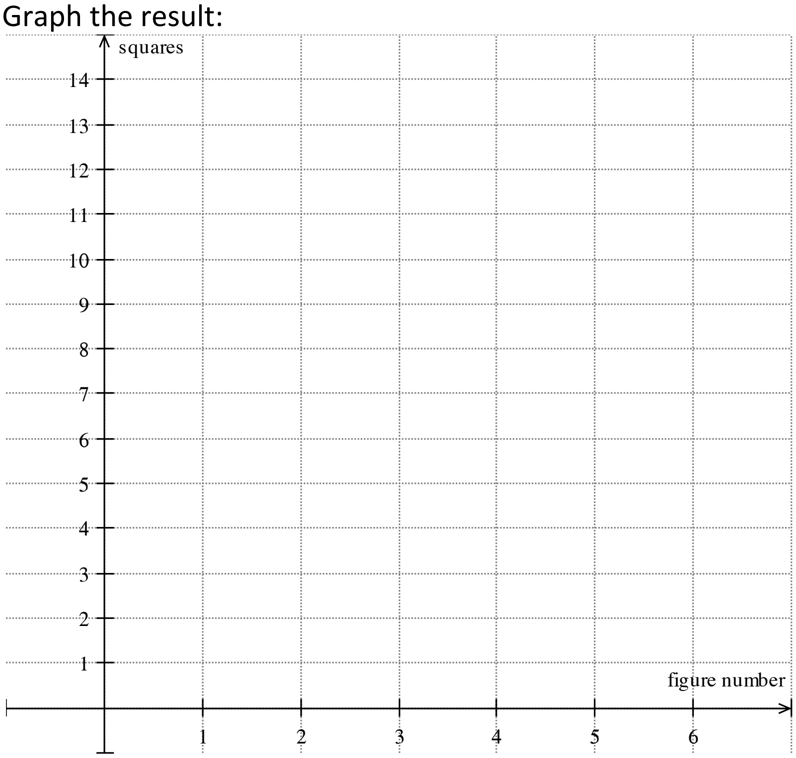


How many little squares are in the 42nd[1] figure?
(FYI: the lone square is the first figure)

Generalize the result: How many little squares are in the *n*th figure?

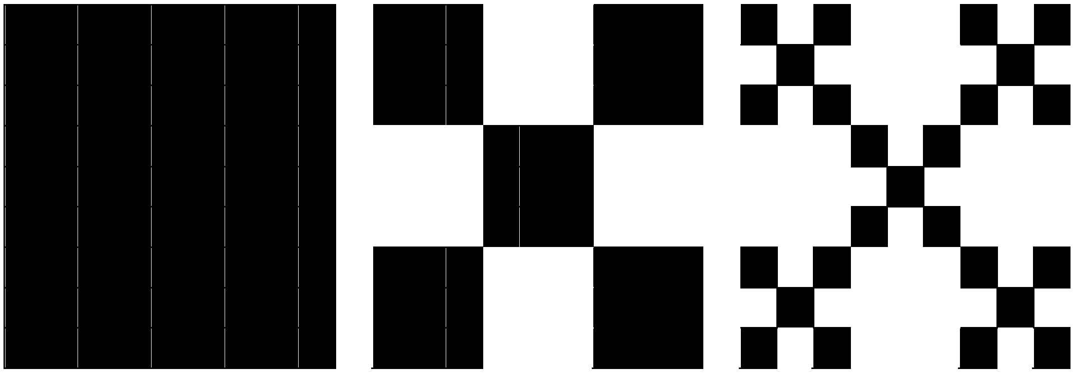
Extend the generalization: How many little squares are in the zero-th figure?

Graph the result:



[1] http://ind.pn/NfegPy

Puzzle #2



Part I: How many squares are in the 42nd figure? (FYI: the first figure has 1 square, the second figure has 5 squares, etc.)

Generalize the result: How many squares are in the *n*th figure?

Part II: If the *length of the square* in the first figure is 3, what is the length of each of the squares in the 42nd figure?

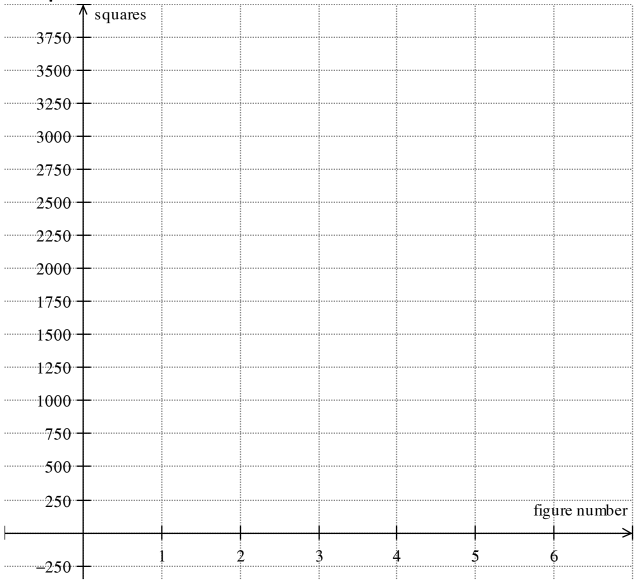
What is the area of one the small squares in the 42nd figure?

How many squares are in the 42nd figure? [You did this on the left hand side!]

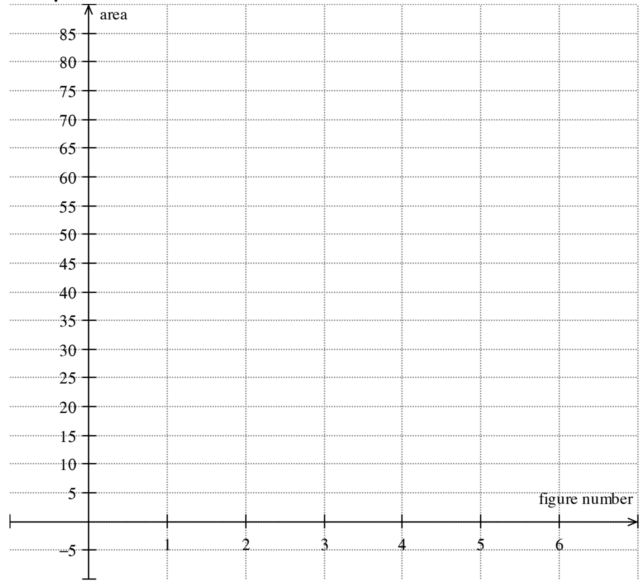
What is the total shaded area of the figure in the 42nd figure?

Generalize the result: What is the area of the *n*th figure?

Graph the results:

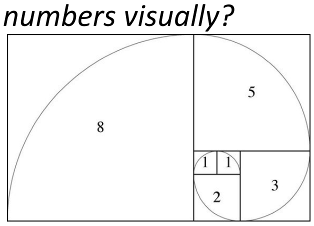


Graph the results:

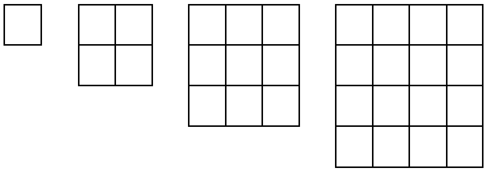


A random comic aside:

How many cheesy tortilla chips are taken in the next two turns? Below is a visualization of that sequence! Can you extend the drawing to see these numbers visually?



Puzzle #3a

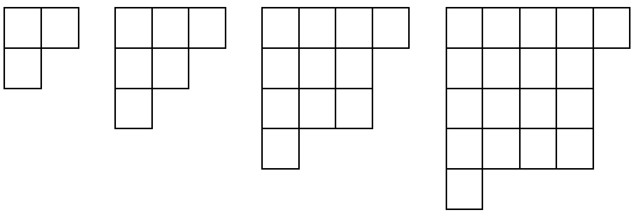


The number of small tiles in the *n*th figure is:

If you had 12 tiles, the largest figure you could build would be the 3rd figure (you don't have enough tiles to build the 4th figure). If you had exactly 7,570 tiles, the largest figure you could build would be the ___ figure.

Explanation:

Puzzle #3b

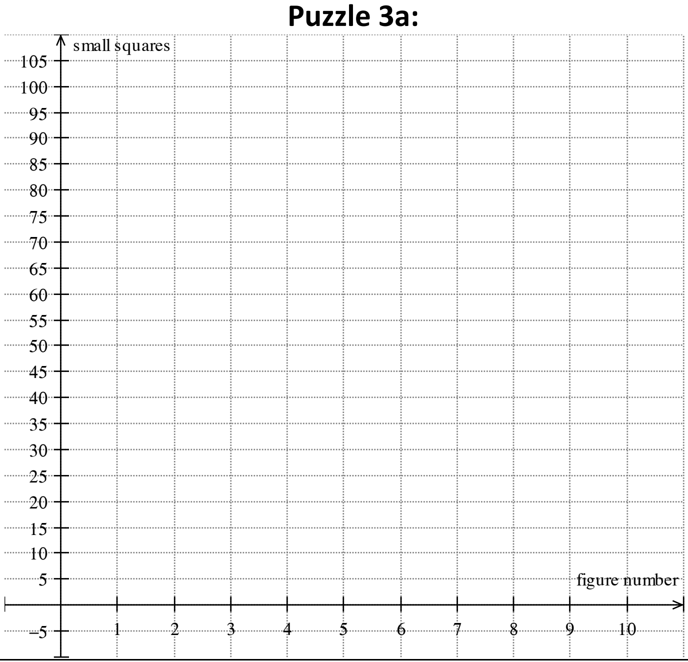


The number of small tiles in the *n*th figure is:

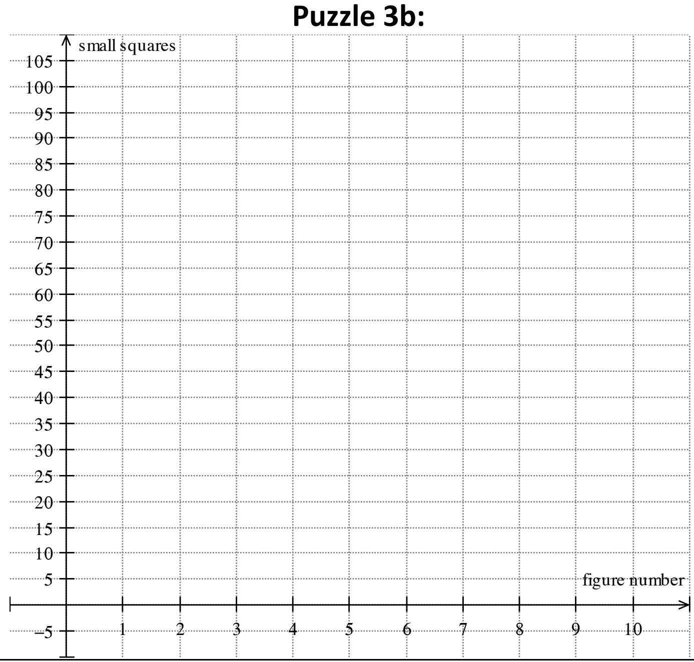
If you had 12 tiles, the largest figure you could build would be the 3rd figure (you don't have enough tiles to build the 4th figure). If you had exactly 7,570 tiles, the largest figure you could build would be the ___ figure.

Explanation:

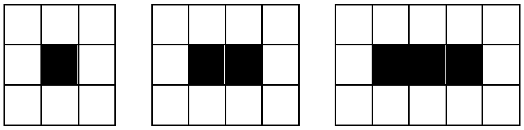
Puzzle 3a:



Puzzle 3b:



Puzzle #4: Gardens are **framed** with a single row of border tiles as **illustrated** here



Draw the 4th garden:

Part I: How many border tiles are required for a garden of *length* 10? (FYI: The length of the first garden is 1.)

Part II: How many border tiles are required for a garden of *length* 30?

Part III: How many border tiles are required for a garden of length 1000? Show and explain how you got your answer.

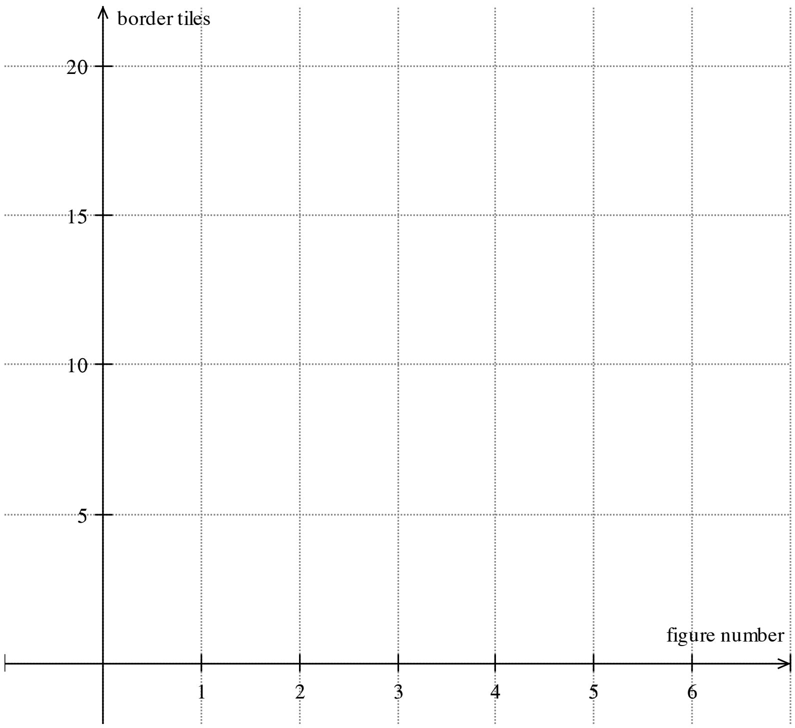
Now that you've found the answer one way, come up with a second (different) way to "count" the border tiles for a garden of length 1000.

Part IV (*generalize the result*): If you know the garden length (call it *n*), explain how you can determine the number of border tiles.

Part V: Show how to find the length of the garden if 152 border tiles are used.

Part VI: Can there be a garden that uses exactly 2012 border tiles? What about exactly 2013 border tiles? Explain your reasoning.

Part VII: *Graph the results*



<u>Part III:</u> Terminology

Each of the puzzles had you generate a set of numbers for the 1st figure, 2nd figure, 3rd figure, 4th figure, etc. In mathematics, we call this a ***sequence***. For example, for Puzzle #1, you saw the pattern 1, 3, 5, 7, …

And we have notation for this. We'll call this sequence $\{R_n\}$ (but we could just as well call it $\{Badger_n\}$ or $\{Snake_n\}$). We use the superextrafancy curly brackets to indicate it's a sequence, and we use the subscript to say where in the sequence we are. So:

Instead of saying…	the 5th number in this sequence R …	**we say** R_5
Instead of saying…	the 27th number in this sequence R …	**we say** R_{27}
Instead of saying…	the *n*th number in this sequence R …	**we say** R_n

As you've seen, the **terms** in a sequence can grow bigger or smaller, and we shall see that they can be crazy and get bigger and smaller and bigger and smaller![2]

Although there are a number of different kinds of sequences (as we shall see), we will really focus on two particular kinds.

In Puzzle #1 and Puzzle #4, we saw the graphs look *linear* and the equation for the *n*th term was a *linear equation.* You can now laugh, because we don't call these sequences linear. We call them ***arithmetic***. That's because arithmetic is about adding and subtracting, and for each term in the sequence we are adding and subtracting a fixed amount. The hallmark of an arithmetic sequence is that there is a **common difference** between each term (if you subtract any term from the previous term, you always get the same common difference).

In Puzzle #2, we saw the graphs look *exponential* and the equation for the *n*th term was an *exponential* equation. You can now laugh again, because we don't call these sequences exponential. We call them ***geometric***, which has something to do with the "geometric mean" (a geometry concept that I am going to ignore here). The hallmark of a geometric sequence is that there is a **common ratio** between each term (if you divide any term by the previous term, you always get the same common ratio).

1. Look at the arithmetic sequence $\{glorph_n\} = \{3,4,5,6,7,8,...\}$
Find what value goes in the square:

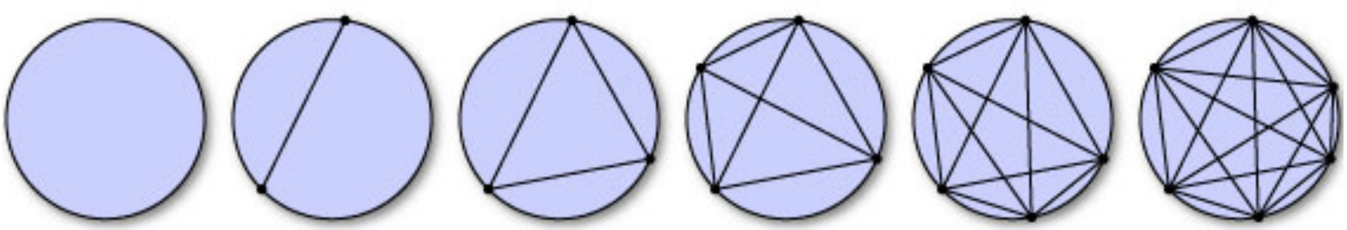
(a) $glorph_{\square} = 4$ □ =_____	(b) $glorph_5 = \square$ □ =_____	(c) $glorph_{\square} = 251$ □ =_____	(d) $glorph_{123} = \square$ □ =_____
In words: "The _____ term in the sequence glorph has a value of 4."	In words:	In words:	In words:

[2] Some sequences are tricky to figure out. Here's a fun one:
$LookAndSay = 1,\ 11,\ 21,\ 1211,\ 111221,\ 312211,...$
Can you figure out $LookAndSay_6$? ______________________________ (solution: http://bit.ly/KBeiSd)

2. Check your understanding of the terminology! Fill in missing information in the table below.

Sequence	**Arithmetic/Geometric/Neither**	**Common Difference/Ratio or Does Not Apply**	**Value of a term**
$\{a_n\} = \{5,4,3,2,1,...\}$		Common Difference is -1	$a_{10} =$
$\{Sam_n\} = \{1,4,9,16,25,...\}$	Neither	Does Not Apply	$Sam_{12} =$
		Common Ratio is -2	$K_5 = -32$
$\{Mattie_n\} = \{1.2,3.2,5.2,7.2,9.2,...\}$			$Mattie_8 =$
$\{L_n\} = \{2,1,3,4,7,11,18,...\}$			$L_{11} =$
		Common Difference is 10.	$T_3 = 5$
$\{Cake_n\} = \{1,2,4,8,16,...\}$			$Cake_6 =$

A not so random aside:
Count the number of regions in each figure. Advice: put a number in each region so you don't double count!

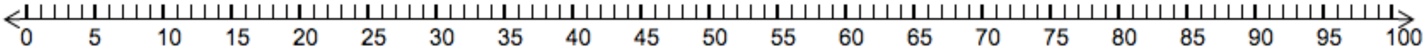


Here are slightly larger versions of the last two figures to help you count!

Part IV: Arithmetic Sequences

1. The first term in an arithmetic sequence (a_1) is 10. The common difference is 3.

(a) On the number line below, draw a dot at $a_1, a_2, a_3, \ldots, a_8$

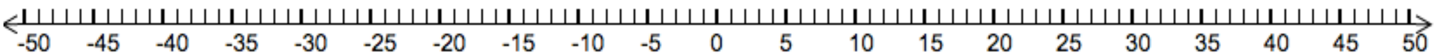


(b) What is the value of a_{12}?

(c) What is the value of a_{71}? What is the value of a_n ?

2. The first term in an arithmetic sequence (a_1) is 13. The common difference is -3.

(a) On the number line below, draw a dot at $a_1, a_2, a_3, \ldots, a_8$



(b) What is the value of a_{12}?

(c) What is the value of a_{71}? What is the value of a_n ?

3. *Generalize!*
(a) Write a formula for the *n*th term in an arithmetic sequence, but instead of using variables, use the terms "the value of the *n*th term," "the value of the first term," and "common difference" in your formula. You may use the variable *n*.

(b) Write that same formula, using the fancy variables: a_n , a_1 , d (for common difference), and *n*.

4. Check your formula with your answer for problems 1c and 2c. Does it work? If not, try to identify where your error lies!

5. (a) You have an arithmetic sequence $\{b_n\}$. If $b_1 = 16.2$ and $d = -10.3$, find b_{2163}.

(b) You have an arithmetic sequence $\{c_n\}$. If $c_1 = \pi$ and $d = 2.3$, find c_{424}.

(c) You have an arithmetic sequence $\{e_n\}$. If $e_1 = 15$ and $d = -25$, find e_{40}.

6. You know the seventh term in an arithmetic sequence is 10. You know the common difference is 5. What is the twenty ninth term in the sequence?

Your challenge is to come up with *two* different ways to get the answer:

Method 1:	Method 2:

7. (a) You know the twelfth term in an arithmetic sequence is -24, and the common difference is -3. What is the sixty first term? What is the fifth term?

(b) What is a formula for the *n*th term in the sequence?

8. (a) You know the nineteenth term in an arithmetic sequence is -50, and the common difference is 8.5. What is the forty second term? What is the fifth term?

(b) What is a formula for the *n*th term in the sequence?

9. You know the twentieth term in an arithmetic sequence is 12. You know the twenty seventh term is 75. What is the common difference?

(a) Your challenge is to come up with *two* different ways to get the answer:

Method 1:	Method 2:

(b) Now find a formula for the *n*th term in the arithmetic sequence.

10. (a) You know the fifteenth term in an arithmetic sequence is 10, and the sixty fifth term is 25. What is the common difference?

(b) Now find a formula for the *n*th term in the arithmetic sequence.

11. (a) You know the five hundred and sixth term in an arithmetic sequence is 2100, and the six hundreth term is -3000. What is the common difference (exactly!)?

(b) Now find a formula for the *n*th term in the arithmetic sequence.

Part V: Geometric Sequences

1. Solve the following. Give both the exact and approximate answers if the answer is "ugly."

$\square^2 = 9$	$\square^2 = -9$	$\square^3 = 8$	$\square^3 = -8$
$\square^2 = 11$	$\square^2 = -11$	$\square^3 = 11$	$\square^3 = -11$
$\square^4 = 341.8801$	$\square^4 = -341.8801$	$\square^5 = 4.48832$	$\square^5 = -4.48832$

2. (a) The first term in a geometric sequence is 5. The common ratio is 2. What is the value of the seventh term?

________	________	________	________	________	________	________
1st term: a_1	2nd term: a_2	3rd term: a_3	…	…	…	7th term: a_7

(b) What is a formula for the *n*th term? (If you need help, figure out the values for the eighth, ninth, tenth, and 11^{th} terms!)

3. (a) The first term in a geometric sequence is 2. The common ratio is -3. What is the value of the ninth term? The tenth term?

(b) What is the formula for the *n*th term?

4. (a) The first term in a geometric sequence is 2. The common ratio is 3. What is the value of the ninth term? The tenth term?

(b) What is the formula for the *n*th term?

5. *Generalize!* (a) Write a formula for the *n*th term in a geometric sequence, but instead of using variables, use the terms "the value of the *n*th term," "the value of the first term," and "common difference" in your formula. You may use the variable *n*.

(b) Write that same formula, using the fancy variables: a_n , a_1 , r (for common ratio), and *n*.

6. Find the value of the eleventh and twelfth terms in a geometric sequence where the first term is 6 and the common ratio is 1/2.

7. Find the value of the eleven and twelfth term in a geometric sequence where the first term is 6 and the common ratio is -1/2.

8. You know the seventh term in a geometric sequence is 49152. The common ratio is 4.
(a) What is the twelfth term? The nineteenth term?

________	________	________	________	________	________	________
7th term: a_7	8th term: a_8	...	...	...	12th term: a_{12}	

(b) What is the fourth term? The first term?

________	________	________	________	________	________	________
1st term: a_1	2nd term: a_2	3rd term: a_3	...	...	...	7th term: a_7

(c) What is the formula for the *n*th term in this geometric sequence?

9. You know the fiftieth term in a geometric sequence is 25. The common ratio is -2.
(a) What is the sixtieth term? The seventieth term?

(b) What is the fortieth term? The first term?

(c) What is the formula for the *n*th term in this sequence?

10. You know the fifth term in a geometric sequence is 12. The common ratio is 2/3.
What is the formula for the *n*th term in this sequence?

11. You know the eleventh term in a geometric sequence is -5. The common ratio is -1/4.
What is the formula for the *n*th term in this sequence?

12. You know the seventy first term in a geometric sequence is 20. The common ratio is 2.
What is the formula for the *n*th term in this geometric sequence? What is the nine hundreth term in this sequence?

13. You have the following terms in a geometric sequence. Determine the common ratio, the value for the 42nd term, the value for the 43rd term, and the value of the *n*th term.

(a) The first term is 10 and the fourth term is 80.

_________ _________ _________ _________

1st term: a_1 2nd term: a_2 3rd term: a_3 4th term: a_4

The common ratio is _____, the 42nd term is _____, the 43rd term is _____, and the *n*th term is _____.

(b): The first term is 12 and the third term is 108.

_________ _________ _________

1st term: b_1 2nd term: b_2 3rd term: b_3

The common ratio is _____, the 42nd term is _____, the 43rd term is _____, and the *n*th term is _____.

Write out the first six terms in $\{b_n\}$:

WAIT A SECOND. Maybe you saw this… maybe you didn't… But there is a second possible sequence for $\{b_n\}$. If you didn't see it, look carefully for a different common ratio that would work.

The common ratio is _____, the 42nd term is _____, the 43rd term is _____, and the *n*th term is _____.

Using this new insight, write out the first six terms in $\{b_n\}$:

(c) Why does part (a) only have *one* possible geometric sequence that works, while part (b) has *two* possible geometric sequences that work?

(d) Why is the following an impossible setup for a geometric sequence?

The first term is 2 and the third term is -128.

_________ _________ __________

1st term: m_1 2nd term: m_2 3rd term: m_3

14. If you know the third number in a geometric sequence is 54 and the fifth number is 486, come up with a formula for the *n*th term. [If there are two possible sequences, say so and show both possible formulas!]

15. If you know the fourth number in a geometric sequence is 156.25 and the ninth number is -488281.25, come up with a formula for the *n*th term. [If there are two possible sequences, say so and show both possible formulas!]

16. If you know the third number in a geometric sequence is 146.88 and the seventh number is 304.570368, come up with a formula for the *n*th term. [If there are two possible sequences, say so and show both possible formulas!]

17. Do you know when there are more than one possible geometric sequences, when you're given two values in the sequence?

If you are given the values of the sixth and tenth terms in a geometric sequence, will you have one or two possible sequences that would match the information given?	
If you are given the values of the fourth and nineteenth terms in a geometric sequence, will you have one or two possible sequences that would match the information given?	
If you are given the values of the eleventh and two hundreth terms in a geometric sequence, will you have one or two possible sequences that would match the information given?	
If you are given the values of the five hundred and forty third and six hundred and first terms in a geometric sequence, will you have one or two possible sequences that would match the information given?	

Part VI: Explicit and Recursive definitions for sequences

In Part I, questions #1-6 gave you a *recursive* definition for a sequence. That means that any term in the sequence was *defined by previous terms*. For example, look at #1:

"1. To get any number in the sequence you add 3 to the previous number. The first number is 10."

Now that we know math notation, we can write this as: $D_n = D_{n-1} + 3$ with $D_1 = 10$.

1. Do the same for the following statements from Part I:

A. To get any number in the sequence, you subtract five from the previous number. The fourth number is 10.

B. To get any number in the sequence, you multiply the previous number by 2 and then add 1. The first number is 4.

C. To get any number in the sequence, you multiply the previous number by 2 and then add 1. The third number is 3.

D. To get any number in the sequence, you add the previous number to twice the number that came before it. The first number is 7 and the second number is 3.

E. To get any number in the sequence, you multiply the previous number by 0.5. The third number is 3.

In Part I, questions #8-15 gave you an explicit definition for a sequence. That means that any term in the sequence was *defined by the term number*. For example, look at #8:

"8. To get the *n*th number in the sequence, you calculate the area of a circle with radius *n*."

Now that we know math notation, we can write this as: $P_n = \pi n^2$.

2. Do the same for the following statements from Part I:

A. To get the *n*th number in the sequence, you raise -1 to the power *n*.

B. To get the *n*th number in the sequence, you multiply *n* by *n-1*.

C. To get the *n*th number in the sequence, you multiply *n-1* by 1.3 and then add 2.

D. To get the *n*th number in the sequence, you raise 2 to the *n-1* power, and then multiply by 3.

E. To get the *n*th number in the sequence, you calculate the number of ways to choose *n-1* objects from *n+1* objects.

Finally, try your hand with recursive sequences! See if you understand them!

3. Here is a recursively defined sequence: $a_n = 2a_{n-1}$ where $a_1 = -3$.
Write in words what the recursive equation is saying:

Find the first six terms:

Is this sequence (circle one): Arithmetic Geometric Neither

4. Here is a recursively defined sequence: $a_n = 2a_{n-1} + 1$ where $a_1 = 5$.
Write in words what the recursive equation is saying:

Find the first six terms:

Is this sequence (circle one): Arithmetic Geometric Neither

5. Here is a recursively defined sequence: $a_n = a_{n-1} - 2$ where $a_1 = 7$.
Write in words what the recursive equation is saying:

Find the first six terms:

Is this sequence (circle one): Arithmetic Geometric Neither

6. Here is a recursively defined sequence: $a_n = a_{n-1} + a_{n-2}$ where $a_1 = 1$ and $a_2 = 1$.
Write in words what the recursive equation is saying:

Find the first six terms:

Is this sequence (circle one): Arithmetic Geometric Neither

7. Here is a recursively defined sequence: $a_n = a_{n-1} + a_{n-2}$ where $a_1 = 2$ and $a_2 = 1$.
Write in words what the recursive equation is saying:

Find the first six terms:

Is this sequence (circle one): Arithmetic Geometric Neither

8. Here is a recursively defined sequence: $a_n = -5a_{n-1} + a_{n-1} \cdot a_{n-2}$ where $a_1 = 2$ and $a_2 = -4$.
Write in words what the recursive equation is saying:

Find the first six terms:

Is this sequence (circle one): Arithmetic Geometric Neither